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# THE CRITICAL SURFACES AND OTHER PROPERTIES OF "SMOOTH" AXISYMMETRIC PLASMA FLOWS IN CHANNELS†

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A stationary axially-symmetric magnetohydrodynamic plasma flow in an azimuthal magnetic field in a plasma accelerator channel is considered. In the "smooth" flow approximation the form of the critical suface (the magnetosonic velocity transition surface) is investigated. It is shown that in a nonpotential flow the formation of bounded supersonic flow regions on a background of subsonic flow is possible. Using an example of potential flow some properties of plasma flows are considered. In particular, the possibility of establishing flows with anomalous density behaviour is demonstrated. Flow in a narrow annular gap of constant size is considered.

## 1. STATEMENT OF THE PROBLEM

WE CONSIDER stationary axially-symmetric magnetohydrodynamic flow in a cylindrical system of polar coordinates $r$, $\varphi$, $z$ in a channel formed by two coaxial electrodes $R_1(z)$ and $R_2(z)$. We shall assume that the velocity has two components $v_r$ and $v_z$ (later denoted simply by $v$) and that the magnetic field has a single azimuthal component $H$. We assume the moving medium to be nonviscous, nonheat-conducting and a perfect electrical conductor. We further assume that the functions $R_1(z)$ and $R_2(z)$ depend only weakly on their arguments, so that one can ignore second derivatives with respect to $z$ and squares of the first derivatives with respect to $z$ ("smooth" flow). With the additional condition of isentropy such a flow is described by the equations [1]

$$\frac{1}{\rho r}\frac{\partial}{\partial r}\frac{1}{\rho r}\frac{\partial\psi}{\partial r}+\frac{\rho r^2}{2}\frac{d\varkappa^2(\psi)}{d\psi}-\frac{dU(\psi)}{d\psi}=0 \tag{1.1}$$

$$\frac{1}{2}\left(\frac{1}{\rho r}\frac{\partial\psi}{\partial r}\right)^2+W(\rho)+\rho r^2\varkappa^2(\psi)=U(\psi) \tag{1.2}$$

$$\frac{H}{\rho r}=\sqrt{4\pi}\,\varkappa(\psi);\quad \frac{\partial\psi}{\partial r}=\rho v r$$

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Here $\psi$ is the hydrodynamic stream function, $W(\rho)$ is the enthalpy, and we shall assume throughout that $W(\rho) = w_0\rho^{\gamma-1}$.

We shall suppose that on the inner electrode $\psi = 0$, and on the outer electrode $\psi = \psi_m$. The $z$ dependence of the unknown quantities remains in the form of a $z$ dependence of the constants of integration of Eqs (1.1) and (1.2).

When investigating MHD flow in a transverse magnetic field there is the interesting question of the transition across the magnetosonic velocity $c_s = \sqrt{\gamma p/\rho + H^2/(4\pi\rho)}$ and of the form of the critical surface at which this transition occurs. The equation of the latter is given by the condition

$$v^2 = c_s{}^2 = (\gamma - 1)W(\rho) + \rho r^2 \varkappa^2 (\psi) \tag{1.3}$$

For brevity we shall later use the term "velocity of sound" to mean the thermal velocity of sound $c_s = \sqrt{\gamma p/\rho}$ in gas-dynamic flow and the magnetosonic velocity in MHD flow. One must similarly interpret the term "supersonic", etc.

Using the Bernoulli integral (1.2) one can find the critical velocity $v_*$ (equal to the local value of $c_s$) on a given streamline in certain special cases:

$$v_*{}^2 = \begin{cases} 2(\gamma - 1)U(\psi)/(\gamma + 1) & \text{for} \quad W \gg \rho r^2 \varkappa^2 \\ {}^2/_3 U(\psi) & \text{for} \quad W \ll \rho r^2\varkappa^2 \quad \text{or for} \quad \gamma = 2 \end{cases} \tag{1.4}$$

Potential flows ($U(\psi) \equiv \text{const}$, $\varkappa(\psi) \equiv \text{const}$) were investigated in detail [1, 2] for the cases $W \gg \rho r^2\varkappa^2$ (ordinary gas dynamics) and $W \ll \rho r^2 \varkappa^2$ (a strong magnetic field or "cold" plasma). It was shown that the critical surface was a plane, and the inclusion of small terms in Eqs (1.1) and (1.2) in the case of strong or weak (but finite) magnetic fields [1, 2], the inclusion of derivatives with respect to $z$ and weak deviation of the function $\varkappa(\psi)$ [1] from constancy distorted the shape of the critical surface, but that these effects were small corrections. Below we shall investigate the problem of critical surfaces in those cases when the deviation from potential flow is not assumed to be small. Furthermore, we shall consider some features of MHD flow which do not take place in ordinary gas dynamics.

## 2. ISOMAGNETIC FLOW

In [1] a flow in which $\varkappa(\psi) \equiv \varkappa_0 = \text{const}$ was called isomagnetic. In this case Eqs (1.1) and (1.2) have the first integral

$$W(\rho) + \rho r^2 \varkappa_0^2 = C(z) \tag{2.1}$$

where $C(z)$ is an arbitrary slowly varying function. Using the Bernoulli integral (1.2) we can write

$$C(z) = U(\psi) - v^2/2 \tag{2.2}$$

If the velocity is known along some streamline, this also determines the function $C(z)$. We shall assume that the velocity $V(z)$ is known along the inner electrode. The acceleration regime ($V'(z) > 0$) corresponds to a decreasing function $C(z)$. It follows from Eq. (2.2) that the velocity increases or deceases simultaneously along all trajectories.

Relation (2.1) determines the implicit function $\rho(r, C(z))$. Substituting it into (1.2) we obtain

$$\int_0^\psi \frac{d\psi}{\sqrt{U(\psi) - C(z)}} = \sqrt{2} \int_{R_1(z)}^r \rho r \, dr \tag{2.3}$$

The function $\rho(r, C(z))$ can, in general, only be found by numerical solution of Eq. (2.1). Cases are, however, possible when it can be expressed explicitly. This significantly simplifies the problem. Suppose that in (2.3) the upper limits of integration are $\psi_m$ and $R_2(z)$, respectively. Then the left-hand integral is a function of $z$ which can be expressed in terms of the function $C(z)$. The right-hand integral is also a function of $z$; it can be represented as a product of two factors, one of which only depends on the function $C(z)$ and the other [which we shall denote by $G(z)$] only depends on the functions $R_1(z)$ and $R_2(z)$. This in fact solves the problem of constructing the shape of the channel from a given velocity at the inner electrode or the inverse problem of finding the velocity along the inner electrode in a channel of given shape. (The inverse problem may, in general, not have a solution, for details see [1].)

Thus, in the following special cases, the function $\rho(r, C(z))$ is expressed explicitly:

$$W(\rho) \gg \rho r^2 \varkappa_0^2, \quad \rho = \left(\frac{C(z)}{w_0}\right)^{1/(\gamma-1)}, \quad G(z) = \frac{R_2^2(z) - R_1^2(z)}{2w_0^{1/(\gamma-1)}} \tag{2.4}$$

$$W(\rho) \ll \rho r^2 \varkappa_0^2, \quad \rho = \frac{C(z)}{\varkappa_0^2 r^2}, \quad G(z) = \frac{1}{\varkappa_0^2} \ln \frac{R_2(z)}{R_1(z)} \tag{2.5}$$

$$\gamma = 2, \quad \rho = \frac{C(z)}{w_0 + \varkappa_0^2 r^2}, \quad G(z) = \frac{1}{2\varkappa_0^2} \ln \frac{w_0 + \varkappa_0^2 R_2^2(z)}{w_0 + \varkappa_0^2 R_1^2(z)} \tag{2.6}$$

Equation (1.3) takes the form

$$v^2 = \alpha C(z) \tag{2.7}$$

where $\alpha = \gamma - 1$ for gas-dynamic flow and $\alpha = 1$ in other cases. It follows from this, using (1.4), that at a sonic transition point on a given streamline the equality

$$C_* = 2U(\psi)/(\alpha + 2) \tag{2.8}$$

is satisfied.

On the other hand, (2.2) implies $C(z) < \min_\psi U(\psi)$. Hence the flow at the entrance will be completely subsonic if

$$U_{\max}/U_{\min} < (1 + \alpha/2) \tag{2.9}$$

We consider, for simplicity, a linear relationship

$$U(\psi) = U_0 + U_1\psi/\psi_m, \quad U_1 > -U_0 \tag{2.10}$$

It follows from condition (2.9) that the following restrictions apply to $U_1$

$$-\alpha/(\alpha + 2) < U_1/U_0 < \alpha/2$$

Substituting expression (2.10) into (2.3) and integrating over the entire section we obtain

$$F(C(z)) = (U_1/\sqrt{2})G(z)/\psi_m \tag{2.11}$$

where

$$F(C) = (\sqrt{U_0 + U_1 - C} - \sqrt{U_0 - C})/C^{1/\alpha}$$

and the function $G(z)$ is determined by formulae (2.4)–(2.6). The extremum of the function $F(C)$ is reached at

$$C_+ = 2\left[2U_0 + U_1 - \sqrt{U_1^2 + \alpha^2 U_0 (U_0 + U_1)}\right]/(4 - \alpha^2)$$

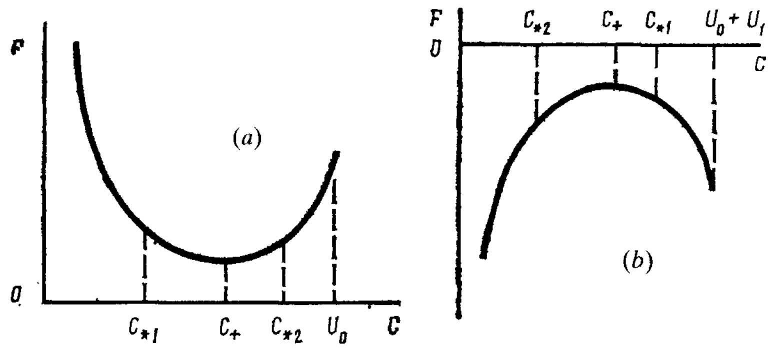


FIG. 1.

($\alpha < 2$) and under the condition of continuous acceleration ($C'(z) < 0$) corresponds to a minimum of the function $G(z)$. One can verify that

$$C_{*1} < C_+ < C_{*2} \text{ for } U_1 > 0; \; C_{*2} < C_+ < C_{*1} \text{ for } U_1 < 0 \tag{2.12}$$

where $C_{*1}$ and $C_{*2}$ correspond to sonic transition points at the inner and outer electrodes and are determined from Eq. (2.8)

$$C_{*1} = 2U_0/(\alpha + 2), \; C_{*2} = 2\,(U_0 + U_1)/(\alpha + 2)$$

Inequalities (2.12) mean that nonuniformity of the distribution $U(\psi)$ leads to distortion of the critical surface. A graph of $F(C)$ is shown in Fig. 1.

Consider the case when $U_1 > 0$ (Fig. 1a). Discontinuous acceleration corresponds to a transition from the ascending branch of the curve $F(C)$ to the descending branch. Here the sonic line (the intersection of the critical surface with the plane longitudinal section of the channel) has the form of the curve $AB$ (Fig. 2a). If $C$ decreases to $C_+$ then the transition to the other branch does not occur and the sonic line has the form of the curve $AC$. When the minimum value of $C$ increases from $C_+$ to $C_{*2}$ the sonic line contracts to the point $D$.

In the case when $U_1 < 0$ (Fig. 1b), as in the previous case, discontinuous acceleration corresponds to a transition from the descending branch of the curve $F(C)$ to the rising branch, and the sonic line has the form of the curve $EG$ (Fig. 2b). If the transition to the other branch of the curve does not occur, the sonic line has the form of the curve $EF$ for the minimum value of $C = C_+$. Increasing the minimum value of $C$ to $C_{*1}$ leads to contraction of the sonic line to the point $H$.

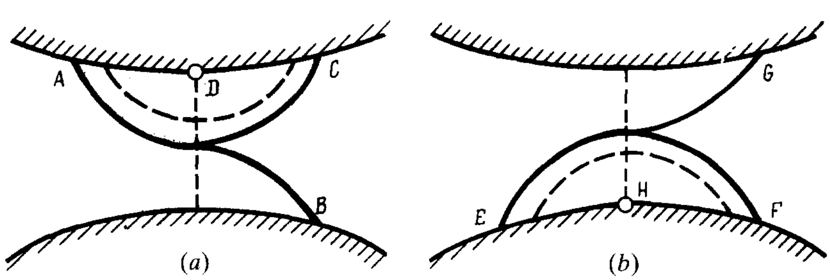


FIG. 2.

We note the following: the strictness of inequalities (2.12) means that the curves $AC$ and $EF$ cannot touch the outer and inner electrodes, respectively.

These discussions have a direct meaning if the shape of the electrode $R_1(z)$ and the velocity $V(z)$ at that electrode are given, i.e. $C(z)$. In this case Eq. (2.11) means that the function $R_2(z)$ can be expressed explicitly in terms of the functions $R_1(z)$, $V(z)$ and the parameters $U_0$, $U_1$, $\varkappa_0$ and $\psi_m$ (i.e. the mass flux). If, however, the geometry of both electrodes is known, together with the freezing-in parameter $\varkappa_0$ and the distribution $U(\psi)$, then (2.11) defines an implicit function $C(z, \psi_m)$. Obviously, the quantity $\psi_m$ cannot be arbitrary, and its maximum value is

$$\psi_+ = D/F\,(C_+),\quad D = (U_1/\sqrt{2})\,\min_z G\,(z) \tag{2.13}$$

For this value of $\psi_m$ the function $C(z)$ at the minimum point of the function $G(z)$ achieves the value $C_+$. Hence (2.13) is a necessary condition for supersonic transition on each streamline. Furthermore, for $\psi_- < \psi_m < \psi_+$, where

$$\psi_- = D/F\,(C_-),\quad C_- = \max\,(C_{*1},\, C_{*2})$$

there exists a supersonic flow domain in the channel which is attached to one of the electrodes, (depending on the sign of $U_1$). For $\psi_m < \psi_-$ the flow is completely subsonic in the channel.

The equations for the critical surfaces $R_*(z)$ in the cases under consideration can be written explicitly. Substituting expression (2.10) into (1.1), we integrate

$$v = V\,(z) + \frac{U_1}{\psi_m}\int\limits_{R_1(z)}^{r} \rho r\,dr \tag{2.14}$$

Note that the direction of increase of the function $U(\psi)$ and the distribution of $v$ in the transverse section are identical. We substitute the expressions for the density from (2.4)–(2.6) into (2.14) and integrate. We then substitute the resulting $v(r, z)$ into (2.7) and find the critical surfaces

$$r_*^2\,(z) = \begin{array}{ll} R_1^2\,(z) + w_0^{1/(\gamma-1)} Q\,(z), & W \gg \rho r^2 \varkappa_0^2 \\ R_1^2\,(z) \exp\,[\varkappa_0^2 Q\,(z)], & W \ll \rho r^2 \varkappa_0^2 \\ [w_0/\varkappa_0^2 + R_1^2\,(z)] \exp\,[\varkappa_0^2 Q\,(z)] - w_0/\varkappa_0^2, & \gamma = 2 \end{array}$$

$$Q\,(z) = \frac{2\psi_m}{U_1}\,\frac{\sqrt{\alpha C\,(z)} - \sqrt{2\,(U_0 - C\,(z))}}{C^{1/\alpha}\,(z)}$$

The formation of bounded domains of supersonic flow in the subsonic flux was observed in calculations of two-dimensional gas-dynamic flow in nozzles [3].

## 3. FLOW IN A STRONG MAGNETIC FIELD

We consider plasma flows for which $W \ll \rho r^2 \varkappa^2$. In this case Eqs (1.1) and (1.2) have a first integral [1]

$$\rho r^2 \varkappa\,(\psi) = C\,(z) \tag{3.1}$$

where $C(z)$ is an arbitrary slowly-varying function. From the Bernoulli integral (1.2) we obtain

$$C\,(z) = [U\,(\psi) - v^2/2]/\varkappa\,(\psi) \tag{3.2}$$

As previously, we determine the function $C(z)$ giving the velocity $V(z)$ along the inner electrode.

The acceleration regime $[V'(z)>0]$ corresponds to a decreasing function $C(z)$. It follows from (3.2) that acceleration or braking occurs simultaneously on all streamlines.

Equation (1.3) in the case undeı consideration takes the form

$$v^2 = \varkappa(\psi) C(z)$$

From this, using the formula for the critical velocity (1.4), we obtain

$$C_* = {}^2/_3 U(\psi)/\varkappa(\psi) \tag{3.3}$$

On the other hand, it follows from (3.2) that $C(z)<\min_\psi\{U(\psi)/\varkappa(\psi)\}$. Hence the flow at the entrance will be completely subsonic under the condition

$$\max_\psi\{U(\psi)/\varkappa(\psi)\}/\min_\psi\{U(\psi)/\varkappa(\psi)\} < {}^3/_2 \tag{3.4}$$

Substituting expression (3.1) into Eq. (1.2) we find

$$\int_0^\psi \frac{\varkappa(\psi)\,d\psi}{\sqrt{U(\psi)-\varkappa(\psi)C(z)}} = \sqrt{2}\,C(z)\ln\frac{r}{R_1(z)} \tag{3.5}$$

For simplicity we put

$$U(\psi) \equiv U_0,\quad \varkappa(\psi) = \varkappa_0 \exp(a\psi/\psi_m) \tag{3.6}$$

Condition (3.4) acquires the form

$$\ln {}^2/_3 < a < \ln {}^3/_2$$

Substituting (3.6) into expression (3.5) and integrating over the entire section, we obtain

$$F(C(z)) = \frac{a}{\sqrt{2}\,\psi_m}\ln\frac{R_2(z)}{R_1(z)}$$

where

$$F(C) = (\sqrt{U_0 - b\varkappa_0 C} - \sqrt{U_0 - \varkappa_0 C})/C^2,\quad b = \exp a$$

The graph of the function $F(C)$ is similar to that shown on Figs 2(a) and (b), its extremum being reached at

$$C_+ = \frac{U_0(b+1)}{2b}\left[1 - \sqrt{1-\frac{32b}{9(b+1)^2}}\right]$$

and under the condition of continuous acceleration $[C'(z)<0]$ corresponds to the minimum point of the function $R_2(z)/R_1(z)$. Investigation shows that

$$C_{*2} < C_+ < C_{*1} \quad \text{for} \quad a>0;\quad C_{*1} < C_+ < C_{*2} \quad \text{for} \quad a<0, \tag{3.7}$$

where $C_{*1}$ and $C_{*2}$ correspond to sonic transition points on the inner and outer electrodes. From (3.3) they are given by the expressions

$$C_{*1} = {}^2/_3 U_0/\varkappa_0,\quad C_{*2} = {}^2/_3 U_0/(b\varkappa_0)$$

Inequalities (3.7) mean that the critical surfaces are distorted.

The analysis of the cases when $a>0$ and $a<0$ is completely similar to the above analysis of isomagnetic flow. An increasing $\varkappa(\psi)$ corresponds to Fig. 2(b) and decreasing $\varkappa(\psi)$ to Fig. 2(a). We note that for fixed channel geometry a necessary condition for a transonic transition on each streamline is $\psi_m = \psi_+$, where

$$\psi_+ = D/F(C_+), \quad D = \frac{a}{\sqrt{2}} \ln \left[\min_z \left\{\frac{R_2(z)}{R_1(z)}\right\}\right]$$

For $\psi_- < \psi_m < \psi_+$, where

$$\psi_- = D / F(C_-), \quad C_- = \max(C_{*1}, C_{*2})$$

there exists a supersonic flow domain in the channel attached to the external electrode in the case when $a<0$ and the internal electrode when $a>0$. For $\psi_m < \psi_-$ the channel flow is completely subsonic. For $\psi_m > \psi_+$, stationary flow, as described by Eqs (1.1) and (1.2), is impossible.

We shall obtain expressions for the velocity $v$. To do this we substitute expression (3.6) into (1.1) and use relation (3.1). Then

$$v = V(z) - \frac{aC^2(z)}{\psi_m} \ln \frac{r}{R_1(z)}$$

We note that the directions of increase of the function $\varkappa(\psi)$ and the velocity distribution $v$ are opposite in the transverse channel section.

## 4. SOME FEATURES OF PLASMA FLOW

Plasma flows in channels have certain properties which can be already seen in the example of potential flow. We recall that in this case, according to (1.1), the $z$-component of the velocity depends only on $z$: $v = V(z)$. In those special cases when the function $\rho(r, z)$ can be expressed explicitly, integration of Eq. (2.3) over the entire section gives

$$\psi_m = Q(V(z))\, G(z), \quad Q(V) = V(U_0 - V^2/2) \tag{4.1}$$

[$G$ and $\rho$ are defined by (2.5) and (2.6)].

1. It follows from expression (4.1) that the critical velocity $V_* = \sqrt{2U_0/3}$ is attained in the section when $G(z) = \min$. We consider the case $\gamma = 2$. Using the fact that $\varkappa_0 = H_0/(\sqrt{4\pi}\rho_0 r_0)$ and $w_0 = 2p_0/\rho_0$, where all quantities with zero indices refer to a point on the inlet section, we can represent $G(z)$ in the form

$$G(z) = \frac{1}{2\varkappa_0^2} \ln \frac{\beta_0 + R_2^2(z)/r_0^2}{\beta_0 + R_1^2(z)/r_0^2}, \quad \beta_0 = \frac{8\pi p_0}{H_0^2} \tag{4.2}$$

The point $z_*$ corresponding to the minimum of the function $G(z)$ is determined by the equation $G'(z) = 0$. If $z_1$ and $z_2$, the extremal points of the functions $R_1(z)$ and $R_2(z)$, do not coincide, then this equation determines an implicit dependence $z_*(\beta_0)$. In other words, the position of the critical surface (in this case a plane) depends not only on the channel geometry, but also on the inlet parameter $\beta_0$. The interval in which the value of $z_*$ can vary is bounded by $z_1$ and $z_2$. In gas-dynamic flow $z_*$ always coincides with the minimum point of the function $R_2^2(z) - R_1^2(z)$ irrespective of whether $z_1$ and $z_2$ coincide or are different.

2. It was shown in [1] that regimes with anomalous density behaviour are possible in MHD theory when the directions of velocity and density increase along the trajectories coincide. In gas-dynamic flow they are always opposite. To be specific we will consider the acceleration regime. For simplicity we put $R_1(z) \equiv R = \text{const}$. Substituting expressions (2.5) and (2.6) into the formula

$$\mathbf{v}\nabla\rho = V(z)\frac{\partial\rho}{\partial z} - \frac{1}{\rho r}\frac{\partial\psi}{\partial z}\frac{\partial\rho}{\partial r}$$

one can find a domain of anomalous acceleration ($\mathbf{v}\nabla\rho > 0$) for a given function $V(z)$. It lies in the subsonic flow and is given by the conditions

$$r^2 > \begin{cases} R^2 P(z), & W \ll \rho r^2 \varkappa_0^2 \\ (\beta_0 r_0^2 + R^2) P(z) - \beta_0 r_0^2, & \gamma = 2 \end{cases}$$

$$P(z) = \exp\{V^2(z)/[U_0 - {}^3/_2 V^2(z)]\}$$

For flow in a channel of fixed geometry we obtain the condition for normal acceleration. The requirement $\mathbf{v}\nabla\rho < 0$ means that

$$V^2(z) > 2U_0\varkappa_0^2 G(z)/[1 + 3\varkappa_0^2 G(z)] \tag{4.3}$$

Because the function $V(z)$ is increasing, while for $z < z_*$ the function $G(z)$ decreases, for normal acceleration throughout the channel it is sufficient that condition (4.3) be satisfied at the inlet section $z = 0$. Finally we represent the sufficiency conditions for normal acceleration in the form

$$A_0^2 > \begin{cases} (1+g)/g, & W \ll \rho r^2 \varkappa_0^2 \\ (1+g)/[(1+\beta_0) g], & \gamma = 2 \end{cases}$$

where $A_0$ is the inlet value of the Alfvén number. In the first case it is taken at an arbitrary point of the inlet section, and in the second, at the same point as $\beta_0$. In these formulae $g = 2\varkappa_0^2 G(0)$.

3. For the case $\gamma = 2$ we consider flow in a narrow annular gap of constant size: $R_2(z) - R_1(z) = H_0 \ll \min_z R_1(z)$. With this condition the function $G(z)$ defined by expression (4.2) takes the form

$$G(z) = (h_0/\varkappa_0^2) R_1(z)/[\beta_0 r_0^2 + R_1^2(z)], \quad r_0 = R_1(0)$$

and the function $Q(z)$ from (4.1) has extrema at the points $z_1$ and $z_2$: $R_1'(z_1) = 0$ and $R_1(z_2) = r_0\sqrt{\beta_0}$. Suppose, to be specific, that $R_1(z)$ has the form shown in Fig. 3. If a subsonic flux is incident on the entrance to such an annular gap, $V(0) < \sqrt{2U_0/3}$, then it will be accelerated when $\beta_0 < 1$, while for $\beta_0 > 1$ it will be slowed down in the interval $z < z_2$ and accelerated in the interval $z_2 < z < z_1$. The sonic transition is possible at the point $z_1$. Figure 3 shows the signs of the velocity derivative at various intervals of the curve $R_1(z)$, the signs under the curve corresponding to transonic flow. At the points $z_2^-$ and $z_2^+$ the velocity has local extrema. Thus, unlike the arbitrary "smooth" flow, for which $0 < V(z) < \sqrt{2U_0}$ [which follows from the Bernoulli integral (1.2)], in a narrow gap of constant dimensions the flow velocity takes values in a narrower range, bounded by the positive roots of the equation

$$V(U_0 - V^2/2) = q$$

where $q = 2\psi_m r_0 \varkappa_0^2 \sqrt{\beta_0}/h_0$, and the quantity $\psi_m$ satisfies the transonic transition condition: $\psi_m = (2U_0/3)^{3/2}/G_{\min}$.

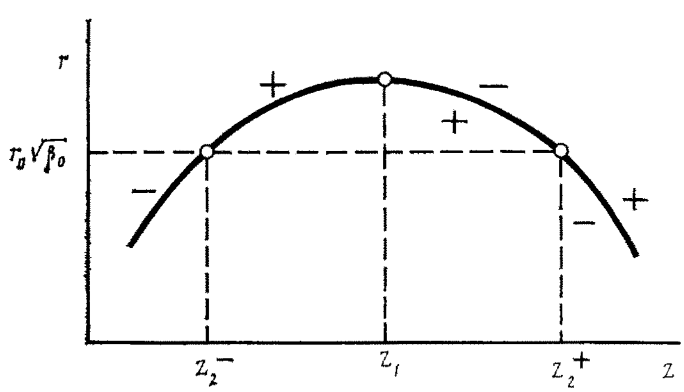


FIG. 3.

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# SHORTWAVE BIRFURCATION IN A MODEL OF A SEISMICALLY ACTIVE MEDIUM AND DOMINANT FREQUENCIES†

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The evolution equations for nonlinear seismic waves possessing a bounded range of frequencies with increasing amplitudes are analysed. It is shown from the evolution equations that the momentum of the system is conserved, and properties of the energy functional are investigated. The spatial period of the mode with the greatest amplification of the initial perturbation is studied. Conservation of convective nonlinearity leads to a stable stationary structure travelling with the velocity of the nonlinear seismic waves.

1. A GENERALIZED model of a visco-elastic body with internal oscillators was proposed [1, 2] for the mathematical study of nonlinear seismic waves. For weak one-dimensional plane longitudinal waves it reduced to a generalized Burgers–Korteweg De Bries equation

$$\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial x} = \sum_{n=2}^{N} (-1)^n A_n \frac{\partial^n v}{\partial x^n} \tag{1.1}$$

where $v$ is the velocity of the displacement and the $A_n$ are positive numbers. Equation (1.1) was obtained by a perturbation method. This equation is general because it was the case $N = 6$ that was considered. Furthermore, the coefficients $A_n$ were chosen so that there existed a range of oscillation

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